Elementary Derivation of Time-Induced Gravitational Mass and Potential Energy

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I. GRAVITATIONAL MASS

One of the most vexing of all questions in physics is what is the origin of mass? Many approaches to this problem have attempted to express the inertial and gravitational mass appearing in Newton's laws of motion in terms of some other more fundamental interaction. While these approaches often appear to explain the origin of mass, they don't explain how or why mass is connected with force.[1–3] General relativity, however, suggests that gravitational forces are manifestations of space-time geometry.[4–10] The following derivation demonstrates a connection between time distortion near a large gravitational body and an effective gravitational mass of matter.

Let us consider a long, essentially massless vessel of length L rigidly attached to a large gravitational source body of mass M. Suppose that wall A of the vessel is facing away from the source body while wall B is in direct contact with the body. Let a photon be emitted at wall A, then travel the length of the vessel, and then be absorbed at wall B. At the moment the photon is emitted at wall A, momentum is imparted to the vessel and the attached body. When the photon is absorbed at wall B, momentum is again transfered to the vessel and the body, such that they are brought to rest. Applying the law of conservation of momentum to the system gives

$$M\mathbf{v} - \frac{h}{c}\left(\nu_B - \nu_A\right)\mathbf{n} = 0 \tag{1}$$

where ν_A and ν_B are the frequency of the photon detected by observers stationed at walls A and B, respectively, h is the Planck constant, $\mathbf{v}$ is the velocity imparted to the vessel and the body, and $\mathbf{n}$ is a unit basis vector normal to wall A. The second term of the left-hand side of (1) suggests that the photon has an effective momentum given by

$$p' = \frac{h}{c}\left(\nu_B - \nu_A\right). \tag{2}$$

Using (2) and expressing the photon momentum also as $p' = m'c$ suggests that the photon possesses an effective gravitational mass given by

$$m' = \frac{h}{c^2}\left(\nu_B - \nu_A\right). \tag{3}$$

Applying the law of conservation of energy to the system suggests that the change in energy of the photon is

$$E' = h\left(\nu_B - \nu_A\right). \tag{4}$$

Comparing (3) and (4) leads immediately to the familiar expression $E' = m'c^2$. Thus, the effective mass and energy of the photon are related to one another just as in the case of ordinary matter. Moreover, it is interesting to note that (3) is entirely gravitationally induced. The effective mass given by (3) is passive gravitational mass since it appears to respond to the gravitational field of M. At the same time, the effective mass of the photon can be viewed as active gravitational mass since it behaves as though it has a gravitational field that attracts M. Thus, the effective gravitational mass of the photon behaves just as does ordinary matter.

It is straightforward to show the connection between the effective mass of the photon given by (3) and time distortion near M. According to observers positioned at wall A, (3) can be recast in terms of proper time as

$$m' = \frac{h\nu_0}{c^2}\left(\frac{d\tau_B}{d\tau_A} - 1\right) \tag{5}$$

where $d\tau_A$ and $d\tau_B$ are intervals of proper time at walls A and B, respectively. Upon expressing (5) in terms of coordinate time and limiting things to first order, the effective mass of the photon assumes the form

$$m' = \frac{h\nu_0}{c^2}\left(\frac{d\tau_B}{dt} - \frac{d\tau_A}{dt}\right). \tag{6}$$

It is easy to see that (6) goes to zero as $d\tau_B \to d\tau_A$, even though $h\nu_0 \neq 0$. This suggests that the difference between $d\tau_A$ and $d\tau_B$ is the primary source of the effective mass of the photon.

II. GRAVITATIONAL POTENTIAL ENERGY

Just as the photon appears to have an effective mass, the photon also possesses a gravitational potential energy. The potential energy of the photon can be determined by noting that if the frequency of the photon detected at wall A is $\nu_A = \nu_0$, then the relative frequency of the photon detected at wall B is

$$\nu_B = \nu_0\sqrt{\frac{1+v/c}{1-v/c}}. \tag{7}$$

Assuming weak field conditions, where $v << c$, we can use the approximation

$$\sqrt{\frac{1+v/c}{1-v/c}} \approx 1 + \frac{v}{c} + \ldots \tag{8}$$

Substituting ν_A and ν_B into (3), and using (8), puts the effective mass of the photon in the form

$$m' = \frac{h\nu_0}{c^2}\left(\frac{v}{c}\right). \tag{9}$$

Upon noticing that we may further approximate things by putting $v = gL/c$, where g is the gravitational field due to M, and using the familiar expression $E' = m'c^2$, (9) leads directly to

$$E' = m_0 gL. \tag{10}$$

Therefore, the gravitational potential energy of the photon is equivalent to that of the mass of the photon m_0 positioned at a height L above the surface of wall B in the presence of the gravitational field g. Just as in the case of the effective gravitational mass (6), the gravitational potential energy (10) is induced entirely by the distortion in time near M. This is most-easily seen by letting $d\tau_B \to d\tau_A$ in (6). Even though $h\nu_0 \neq 0$, setting $d\tau_B = d\tau_A$ puts both (6) and (10) equal to zero.

It is interesting to note that observers residing on M cannot see the photon, and thus may ascribe the effective mass (6) and the gravitational potential energy (10) directly to the vessel. Moreover, were the vessel very small in size, observers on M might be inclined to view the vessel as an ordinary particle of matter. If the energy comprising ordinary matter responds to distortions in time in the same way as does a radiation-filled vessel, then it is not too difficult to propose that time distortion is the origin of the mass appearing in Newton's laws of motion.

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